Part I

Total 61 points

(4+6)

1.

(a) Give an example of unambiguous CFG G which has two distinct derivations for some string in L(G). Write those derivations, too.

(b) Give an example of ambiguous type-3 grammar. Show that it is really ambiguous. Write also an equivalent unambiguous type-3 grammar.

(6)

2. What is the language generated by each of the following grammars?

1) $G_1 = (\{S\}, \{a,b\}, \{ S \rightarrow SSSSS \mid ab \mid b \}, S)$

2) $G_2 = (\{S,A\}, \{a,b,c\}, \{S \rightarrow AbScA \mid \varepsilon,\ A \rightarrow aA \mid bA \mid a\}, S)$

3. Construct a pushdown automaton **with one state** which accepts the language $\{ ww^R \mid w \in \{a,b\}^* \}$ by empty stack.

(6+6)

4. What are the languages generated by the following grammars? Prove that they are ambiguous, and write an equivalent unambiguous grammar.

1) $G_1 = (\{S,A,B,C,D\}, \{a,b\}, P_1, S)$, where

$P_1 = \{ S \to AB,\ A \to aA \mid aC,\ B \to Bb \mid Db,\ C \to Cb \mid \varepsilon,\ D \to aD \mid \varepsilon \}$

2) $G_2 = (\{S\}, \{a,b\}, \{ S \to Sb \mid aSb \mid aaSb \mid \varepsilon \}, S)$

(5)

5. Suppose that G is a CFG and that w, of length m, is in L(G).

1) How many steps does a derivation of w in G take if G is in CNF?

2) How many steps does a derivation of w in G take if G is in GNF?

(3+4+5)

6. Write a context-free grammar for each of the following languages.

1) $\{ bwaw^Ra \mid w \in \{a,b\}^* \}$

2) $\{ a^i b^j c^k \mid i < j+k,\ \ i,j,k \in \mathbb{N} \}$ with $V=\{S,X\}$

   where the language of X is $\{ a^m b^n \mid m < n,\ \ m,n \in \mathbb{N} \}$

3) $\{ a^m b^n c^p d^q \mid m+p = n+q,\ \ m,n,p,q \in \mathbb{N} \}$ (Read carefully.)

(6+1+3)

7. Let $L = \{ w \in \{a,b\}^* \mid |w|_a = |w|_b \}$, $L_a = \{ w \in \{a,b\}^* \mid |w|_a = |w|_b + 1 \}$, and $L_b = \{ w \in \{a,b\}^* \mid |w|_b = |w|_a + 1 \}$.

(1) Prove that if $x \in L_a$ and $x = by$ for some string y, then $y = wz$ for some strings $w, z \in L_a$, using $Suf = \{ v \mid v$ is a suffix of y and $|v|_a > |v|_b + 1 \}$.

(2) If $x \in L$ and $x=by$, what can you say about y?

(3) **Using** (1), the corresponding result for L_b, and (2), find a CFG generating L that has exactly three variables E, A, B, generating L, L_a, and L_b, respectively.